Stochastic Individual Buy Model (Lifetime buys)

This document describes a simple quantity prediction model that computes probability distributions of buy quantities for individual part lifetime or bridge buys, buy sizes that satisfy a specified confidence level and the probability of being overbought or underbought by a user specified quantity.

This model performs the following:

- Computes probability distributions of buy quantities for individual part lifetime or bridge buys
- Computes buy sizes that satisfy a specified confidence level
- Computes the probability of being overbought or underbought by a user specified quantity

The inputs to the model are:

- Length of time you are buying for (in time periods)
- Demand forecast in each time period (this can be correlated period to period)
- Length of time needed to design out the part or identify another solution (if necessary)
- Desired confidence level

All of the input quantities can be entered as probability distributions.

The outputs from the model are:

- Buy quantity as a probability distribution
- Buy quantity that satisfies confidence level

Example Results[1]

Figure 1 shows an example output from the model. The inputs for this example are given

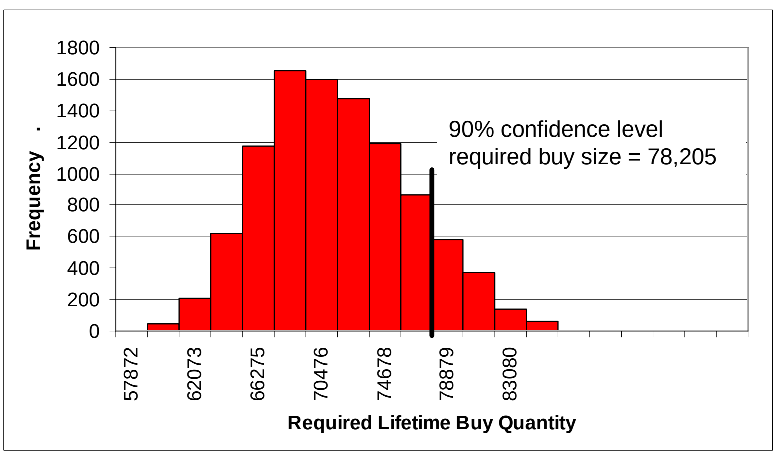

Figure 1 – Probability distribution of required lifetime buy quantity.

[1] Note, this tool performs a stochastic analysis, therefore, every time you run the tool you will get slightly different answers (e.g., you will get slightly different answers for this example as well.

Table I – Example case inputs

Time Period	Distribution Type	Mode (None, Triangular, Normal)	Standard Deviation (Normal)	Low (Uniform, Triangular)	High (Uniform, Triangular)	Correlation Coefficient to Previous Period
1	Normal	11000	750	8500	13500	0
2	Normal	11000	750	8500	13500	0
3	Uniform	11000	750	8500	13500	0
4	Normal	11000	750	8500	13500	0
5	Uniform	11000	750	8500	13500	0
6	Uniform	11000	750	8500	13500	0
7	Uniform	11000	750	8500	13500	0
8	Uniform	11000	750	8500	13500	0
9	Uniform	11000	750	8500	13500	0
10	Uniform	11000	750	8500	13500	0

The length of the buy = 6 time periods
Length of the redesign out (time periods) = 0 to 7.5 (mode = 6) triangular distribution

in Table I (note, all the correlation coefficients in the last column of Table I are 0 indicating that there is no correlation between the sampled demand quantities from period-to-period). The plot shows a histogram of all the required lifetime buy quantities based on the inputs and their associated uncertainties. In this case, the mean buy quantity is 71,526. The user specified a desired 90% confidence level that they bought a sufficient quantity of parts, which corresponds to 78,205 parts in this case. The confidence level is the confidence that you will have enough parts (confidence that you have not underbought), e.g., 0.9 represents a 90% confidence that you will not run out of parts. In other words, given the uncertainties, 90 out of 100 times, you will have enough parts; 10 or of 100 times you will run short.

The confidence level says nothing, however, about the degree to which you will be overbought or underbought. The model also computes the probability of being overbought or underbought by a user specified quantity. Figure 2 shows that there is a 64% probability that buying 78,205 parts will result in having a surplus of 5000 parts.

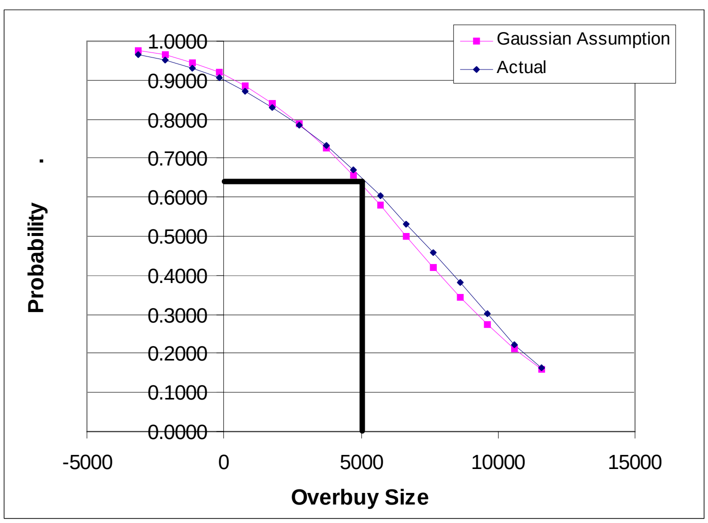

Figure 2 – Probability being overbought or underbought (a negative overbuy size indicates an underbuy).

The overbuy size vs. probability graph should be interpreted the following way. For a given confidence level, assuming that you bought the quantity given corresponding to the confidence level, the graph provides the probability of a specific overbuy or underbuy size. For a given overbuy size (on the horizontal axis, e.g., 5000), the Probability is the probability of the overbuy being greater than or equal to 5000 for the confidence level specified. Negative overbuys are underbuys. So, in Figure 2, the probability that you will be overbought by 5000 or more parts is 0.64 (64%), similarly, the probability that you will be underbought by 3200 or less (in this case less includes being overbought by any amount) is 0.98. Two sets of data are plotted on the graph. The "Gaussian Assumption" data is based on an assumption that the lifetime buy quantity histogram can be approximated as a normal distribution. The "Actual" data is generated without making any assumptions about the shape of the lifetime buy quantity histogram.

Running the Model

This application is for use with Excel (developed using Microsoft Office Excel 2003)

The spreadsheet uses Macros that must be enabled. You can enable the Macros buy opening Excel and selecting Tools->Options…, Security tab, Macro Security…, choose Medium (or Low); restart the spreadsheet (select "Enable Macros" in the Security Warning if asked).

Basic Model

This model implements the following algorithm:

$$\text{Lifetime buy quantity} = \sum_{i=1}^{\lfloor L \rfloor} Q_i + \left(L - \lfloor L \rfloor\right) Q_{\lfloor L \rfloor + 1} \qquad (1)$$

where

Q_i = marketing recommended buy quantity in time period i (all periods are assumed to have the same length)

L = length of the buy in time periods, i.e., the number of time periods until the part is no longer needed

$\lfloor \; \rfloor$ = floor function (round down to the nearest integer).

The first term in (1) sums the recommended buy quantity over whole time periods. The second term in (1) accounts for a fractional time period. This model assumes that redesigns (if any) are initiated a sufficient duration prior to L in order to be completed at L.

Stochastic Modeling

Both the Q_i and L terms in (1) are uncertain (in addition to the length of the redesign). Sample values of Q_i and L are generated and used in (1) to compute sample lifetime buy quantities. Sampling and calculation of lifetime buy quantities is repeated many times to generate a histogram of lifetime buy quantities using a Monte Carlo sampling approach. The value of Q_i is characterized as a probability distribution (which can be different for each time period i). The sampled length of the buy (L_s) is computed using the following relation,

$$L_s = L_{b_s} - \left(L_r - L_{r_s}\right) \qquad (2)$$

where

L_{b_s} = Sampled forecasted length of buy in time periods

L_r = Length of the redesign (mode) in time periods (planned length of redesign)

L_{r_s} = Sampled length of the redesign in time periods (actual length of redesign).

Equation (2) assumes that the redesign starts L_r (mode) time periods before L_b (mode) and that the actual end of the redesign (the point when you don't need any more parts) is given by (2). Both L_r and L_b are characterized as probability distributions. If there are no uncertainties, then L_s in (2) is L_b (this assume that the redesign took exactly the amount of time it was forecasted to and that the forecasted length of the buy was exactly correct).

Note, the analysis approach was specifically formulated so that the length of the periods is constant (i.e., the length of the periods cannot be uncertain), but the number of periods can be uncertain – this was done so that the length of the buy can be treated as independent of the quantities in each period. If simple random sampling is used, the above analysis approach is correct if the underlying distributions are independent. However, the quantities from period to period are probably not independent. Therefore, we have provided a correlation coefficient input associated with the quantities.[2] The correlation coefficient can vary from -1 to 1 (inclusive). Setting the correlation coefficient to 0 indicates that there is no correlation with the previous period's quantity (the distributions are independent).

Distribution Assumptions

The types of distributions that can be chosen are: None = no distribution, mode is used; Uniform = range from the low value to the high value used (all values in the range are equally likely); Triangular = starts at low value, ends at high value, mode is the peak; and Normal = characterized by the mode (mean in this case) and a standard deviation. The post processing analysis (confidence level and overbuy/underbuy probability) makes no assumptions about the shape of the resulting histogram of lifetime buy quantities.

Confidence Level

The confidence level is the confidence that you will have enough parts (confidence that you have not underbought), e.g., 0.85 represents an 85% confidence that you will not run out of parts. In other words, given the uncertainties, 85 out of 100 times, you will have enough parts; 15/100 times you will run short. This confidence level says nothing about the degree to which you will be overbought or underbought.

[2] Correlation coefficient is a measure of the degree of the relation between two sets of data. The correlation coefficient varies between -1 and 1. Positive correlations (between 0 and 1) indicate that high values in one set of data are related to high values in another set of data. Negative correlations (between -1 and 0) indicate that high values in one set of data are related to low values in another set of data.

Known Problems

- Due to the fact that normal distributions range from $-\infty$ to $+\infty$, sampling from normal distributions can produce negative numbers that do not represent physically realistic quantities. At the present time, if a negative number is returned, it is replaced with zero. Technically this *ad hoc* truncation means that the normal distributions used in this analysis are not actually normal and not actually even valid probability distributions. A truncated normal distribution (truncated at zero after shifting) should be used.
- Normal distributions cannot be correlated in the present implementation, i.e., the quantity in the period after a normal distribution cannot be correlated to and a normal distribution cannot be correlated to a previous period.
- The implementation of correlated sampling (to accommodate the correlation coefficients) is rather *ad hoc*. It's probably more accurate to describe it as an approximation to correlation coefficients.

Peter Sandborn
CALCE, Department of Mechanical Engineering
University of Maryland
(301) 405-3167, sandborn@calce.umd.edu